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ELECTRON DRIFT IN A LINEAR MAGNETIC WIGGLER WITH AN AXIAL GUIDE--ETC(U)
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Electron trajectories through a linear magnetic wiggler in an axial guide field are calculated numerically. Off-axis electrons are shown to drift in a direction perpendicular to the wiggler field because of the wiggler gradient. Effects of self fields and initial conditions are analyzed, and the results are compared to those obtained with a helical wiggler. An empirical analytic expression for the linear wiggler drift is presented, and means of stabilizing the drift are discussed.

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ELECTRON DRIFT IN A LINEAR MAGNETIC WIGGLER WITH AN AXIAL GUIDE FIELD

Recently, there has been much interest in free electron lasers (FEL's) which use spatially varying magnetic fields to modulate a relativistic electron beam (REB).[1-5] Three types of magnetic wigglers have been widely used: helical (usually produced by a bifilar helical current winding[6,7]), linear (produced by an array of permanent magnets[8] or by linearly alternating windings[9]), and radial[10] (produced by a series of spaced conducting or ferromagnetic washers[11] immersed in an axial field or by a series of alternating coils[3]). Of these, only the first two produce a perpendicular field on axis and are therefore suitable for use with a small diameter beam of solid cross-section. In this paper, we will analyze the propagation of a solid, high current REB through linear and helical wigglers with a superimposed axial guide field.

FEL experiments fall into two categories depending on the beam current. Compton regime FEL's[1,2,5] have used high energy (10's of MeV), low current ($\lesssim$ 1 A) beams while Raman FEL's [3,4] have used lower energy ($\sim$ 1 MeV), high current ($\gtrsim$ 1 kA) beams. Helical wigglers have been used in both current regimes, but until recently[9] linear wigglers were used only in the Compton regime. In this case, the linear wigglers consist of permanent magnets and there is no guide field. However, for high current beams, a guide field is required to contain the beam. Although there are advantages to a linear wiggler from the standpoint of ease of assembly and versatility (e.g., changing the periodicity or tapering the period and/or field amplitude for efficiency enhancement[12]) we will show that there is no equilibrium for off-axis particles when a beam is propagated through a linear wiggler in an axial field. The particles drift out of the wiggler unless additional focusing forces are provided.

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Various authors have considered particular cases of electron motion through magnetic wigglers. These have typically been single particle (low current) calculations. Blewett and Chasman[7] considered motion of high energy electrons (~ 24 MeV) through a helical wiggler and found stable helical orbits with superimposed betatron oscillations. Friedland[13] has treated the case of lower energy electrons (~ 300 keV) in an idealized radially uniform helical wiggler with a superimposed axial guide field. He showed that various stable trajectories were possible and derived stability criteria relating the allowed wiggler and guide field strengths to beam energy and wiggler period λ_w. These "stable" regions are given by

$$\frac{B_w}{B_o} < \left[\left(\frac{vk}{\Omega_o}\right)^{2/3} - 1 \right]^{3/2} \tag{2a}$$

and

$$\Omega_o > ck, \tag{2b}$$

where B_o is the axial guide field, $\Omega_o = eB_o/\gamma m$ is the corresponding cyclotron frequency, v is the electron velocity, and $k = 2\pi/\lambda_w$. Physically, these conditions stem from the resonance in the perpendicular velocity of an electron in the wiggler and guide fields:[4]

$$v_w = - \frac{v_z \Omega_o}{v_z k - \Omega_o} . \tag{3}$$

Freund and Drobot[14] have considered this case further and also conclude that stable trajectories with nearly constant axial velocities and relatively large

wiggler amplitudes are possible when $\Omega_o << ck$. This is consistent with condition (2a).

The present analysis employs a relatively simple computer code which solves the equations of motion of an electron in any electric and magnetic field configuration using a fourth-order Runge-Kutta method. Self electric and magnetic fields are calculated by assuming that the electron is at the edge of an azimuthally symmetric beam of current I, so that the self fields can be written as (mks)

$$E_r = \frac{-I}{2\pi\varepsilon_o r v_z}$$

$$B_\theta = \frac{-\mu_o I}{2\pi r} , \tag{4}$$

where r(t) is the electron radius. The axial self fields are neglected.

For the particular cases considered here, the external magnetic field consists of a solenoidal field produced by a 15.3-cm-I.D. x 2-m-long solenoid together with a wiggler that begins in the uniform portion of the solenoidal field. The wiggler field, which may be either helical or linear, rises adiabatically over ten periods and then oscillates with constant amplitude. The envelope enclosing the wiggler amplitude is given by

$$b(z) = \begin{cases} \frac{1}{2}\left[\left(\frac{z}{10\lambda_w}\right)^2 + \left(\frac{z}{10\lambda_w}\right)^3\right], & 0 < z < 10\ \lambda_w \\ 1 & , z > 10\ \lambda_w . \end{cases} \tag{5}$$

Here, z is the axial distance from the beginning of the wiggler. This variation closely fits the linear wiggler used in the NRL induction linac FEL. The field on axis from this wiggler is plotted in Fig. 1 along with the envelope equation (5).

The linear wiggler field components are

$$\begin{aligned} B_x^{\ell} &= b(z) B_w \cosh kx \cos kz \\ B_y^{\ell} &= 0 \\ B_z^{\ell} &= -b(z) B_w \sinh kx \sin kz. \end{aligned} \tag{6}$$

where B_w is the peak wiggler field on axis. For the NRL linear wiggler,[9] which has $\lambda_w = 3$ cm and winding layers spaced by 3.2 cm, these expressions are valid for $r \lesssim 1$ cm.

We will be primarily concerned with relatively low energy (~1 MeV), high current (~kA) beams. Of particular interest are the effects of self-fields, guide field and wiggler amplitude, and initial beam conditions corresponding to a field-immersed ($v_{\theta o} = 0$) or a shielded source ($P_{\theta o} = 0$), where $v_{\theta o}$ is the intitial azimuthal electron velocity and $P_{\theta o}$ the initial canonical angular momentum. If the axial self magnetic field is neglected, $P_{\theta o} = 0$ implies $v_\theta = erB_z/(2\gamma m)$ in a uniform field B_z.

First, we consider a beam with $\gamma = 2.2$, $I = 250$ A, $B_o = 2$ kG, $B_w = 1$ kG, and $\lambda_w = 3$ cm. Note that Eq. (2a) would require $B_w < 3$ kG for stability in this case with a helical wiggler. The x-y trajectories of electrons injected

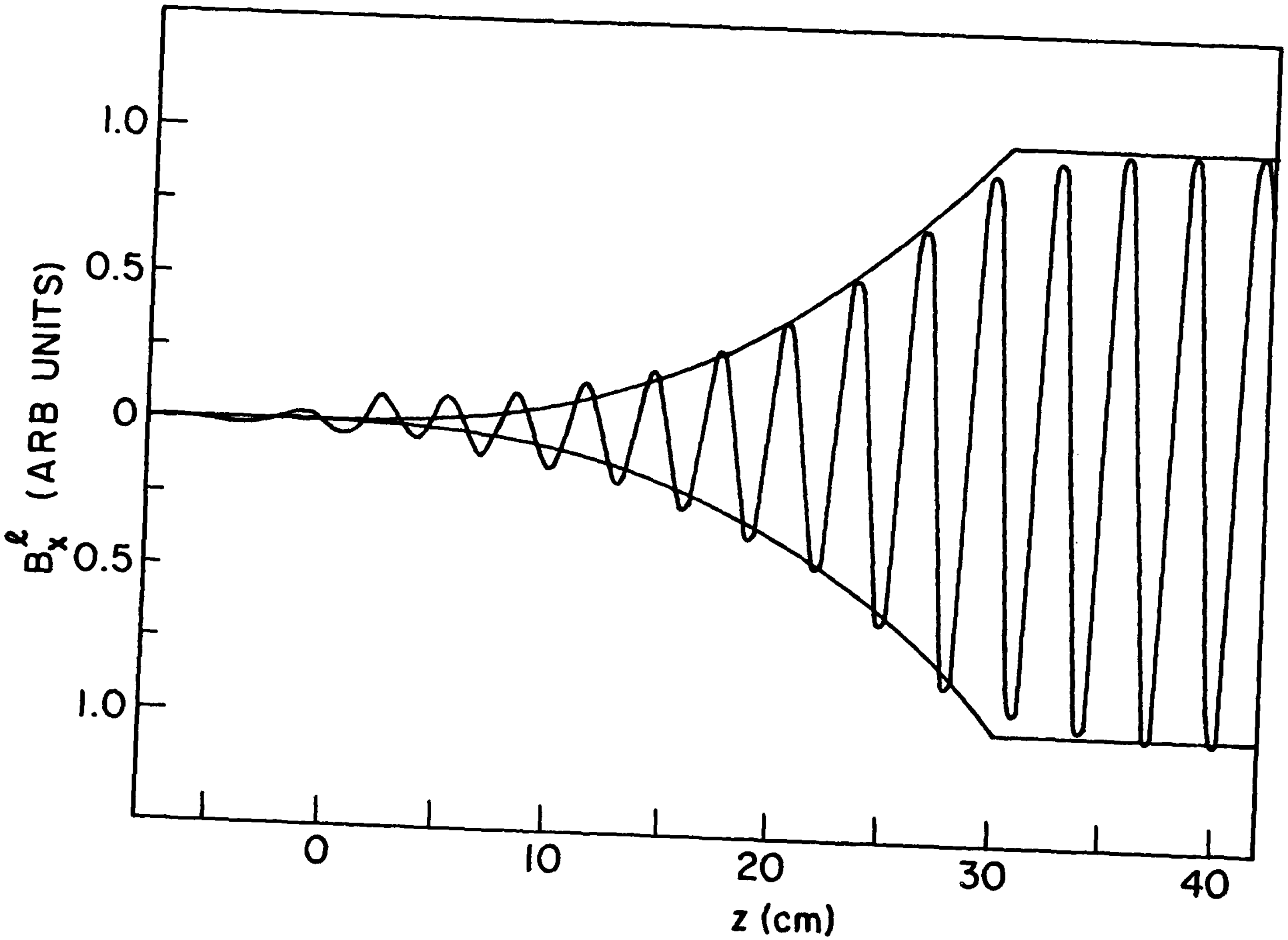

Fig. 1 - Plot of measured $B_x^\ell(z)$ from NRL linear wiggler together with envelope $b(z)$ from Eq. (5)

with $v_{\perp o} = 0$ and $r_o = 0.4$ cm into a linear wiggler are shown in Fig. 2. Each particle initally begins to $\vec{E} \times \vec{B}$ drift in the self fields, but when it reaches a region of large wiggler field it begins drifting in the y-direction. In each case the electron reaches an imaginary wall, located at r = 1.2 cm, after only partially traversing the wiggler. It should be noted that the assumptions used to calculate the self fields become invalid as the beam distorts. Consequently, to determine if these self fields are responsible for this drift, the calculations are repeated with I = 0 (Fig. 3). A third trajectory is also plotted for $B_w = 0.5$ kG. The electron that originates on the y-axis is well confined, but electrons off the y-axis again drift to the wall, with a velocity much higher for $B_w = 1$ kG ($\langle v_d \rangle \approx .047$ c) than for $B_w = .5$ kG ($\langle v_d \rangle \approx .011$ c).

This behavior can be explained by the increasing gradient in the wiggler field, and consequently the emergence of a significant axial field component, as the distance from the y axis increases. Although the present drift arises from the gradient in B_w and is in the direction of $\nabla B_w \times \vec{B}_o$, it is quantitatively different from the usual guiding center approximation because the field variation over one gyroperiod is so extreme. For example, when kx = .8 (x ≈ .4 cm in the present case), B_z^{ℓ} varies from + B_w to − B_w over one period. However, the physical mechanism is the same as for the usual gradient drift; i.e., the gyroradius in the part of the orbit where B_z is a minimum, or where $\|x\|$ is a minimum in Fig. (3), is larger than where B_z is a maximum. The addition of self fields merely imposes an additional $\vec{E} \times \vec{B}$ rotation on the obit, so that an electron that originates on the y-axis (and is therefore confined when I=0) begins to drift into a region of increasing B_z^{ℓ}. Therefore, it actually is "lost" sooner than an electron on the x-axis which

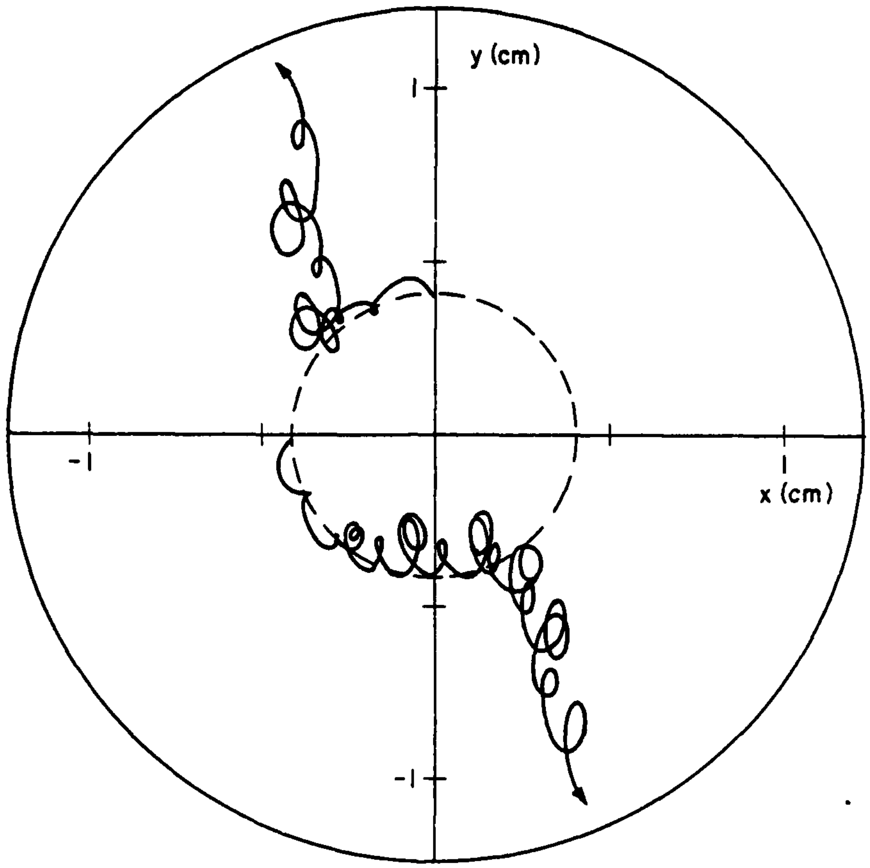

Fig. 2 - Electron trajectories in linear wiggler with λ_w = 3 cm, B_o = 2 kG, γ = 2.2, r_o = .4 cm, I = 250 A, and B_w = 1 kG.

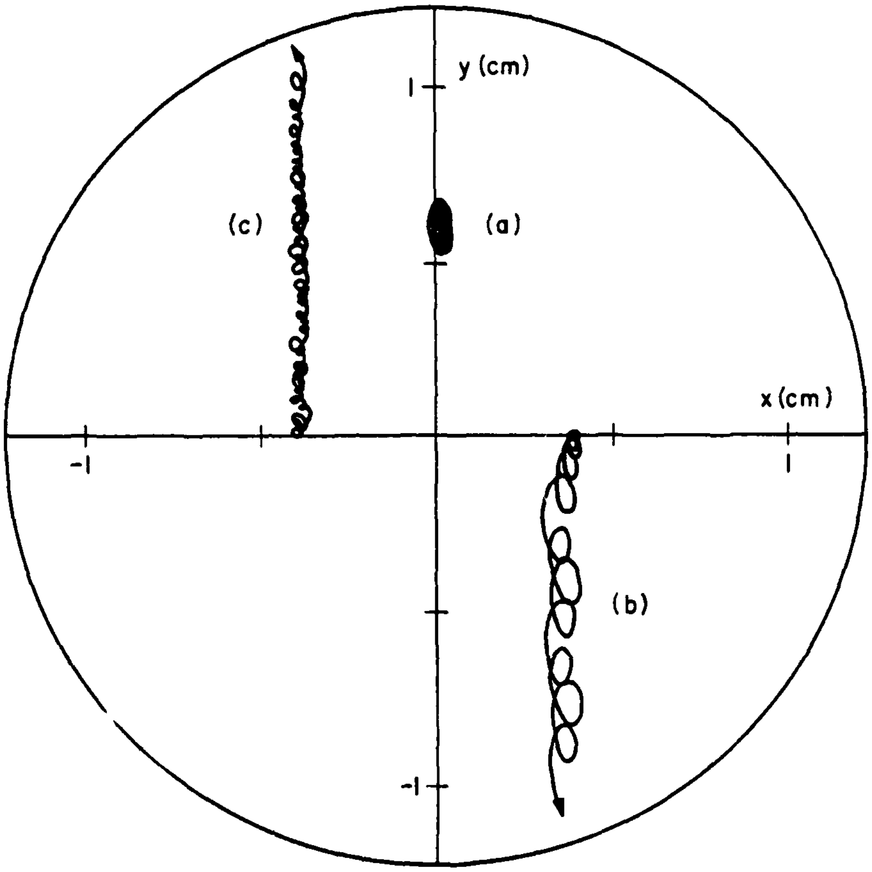

Fig. 3 - Electron trajectories in linear wiggler with λ_w = 3 cm, B_o = 2 kG, γ = 2.2, r_o = .4 cm, and I = 0. B_w = 1 kG for particles a and b, and B_w = .5 kG for particle C.

initially $\vec{E} \times \vec{B}$ drifts into a region of smaller B_z^ℓ .

An approximate, empirical expression for the drift has been found which is in quite good agreement with the code results when $\Omega_{wz} < \Omega_o < kv_z$ (below the cyclotron resonance). In terms of the wiggler gradient, the expresson is

$$\vec{v}_d \approx \frac{1}{2} \frac{v_\perp^2}{\Omega_o} \frac{\nabla (B^\ell)^2}{2(B^\ell)^2} \times \frac{\vec{B}_o}{B_o} , \tag{7}$$

which has the same form as the usual grad B drift but is quantitatively different. The single particle equations of motion are $\dot{v}_x = v_y \Omega_z$, $\dot{v}_y = - v_z \Omega_x + v_x \Omega_z$, and $\dot{v}_z = v_y \Omega_x$. Assuming $v_z \approx$ constant and that the gyration velocities are $v_y = v_{\perp y} \sin kz$ and $v_x = v_{\perp x} \cos kz$, it follows that

$$v_{\perp y} \approx v_z \frac{kv_z \Omega_w \cosh kx}{\Omega_o^2 - k^2 v_z^2}$$

$$v_{\perp x} \approx v_z \frac{\Omega_o \Omega_w \cosh kx}{\Omega_o^2 - k^2 v_z^2} \tag{8}$$

for $B_x^\ell \gg B_z^\ell$. These assumptions are reasonably valid for $kx \lesssim .8$ and for B_o sufficiently far from resonance and B_w small enough that $v_\perp \ll v_z$. Furthermore, to insure that cosh kx $\approx$ constant over the orbit, we restrict $v_{\perp x} \ll v_{\perp y} \approx v_\perp$ (i.e., $\Omega_o < kv_z$). Then using Eqs. (8) and (6) in Eq. (7), we obtain

$$v_d \approx \frac{1}{2} v_z \frac{kv_z}{\Omega_o} \frac{(\Omega_w kv_z)^2}{(\Omega_o^2 - k^2 v_z^2)^2} \cosh kx \sinh kx. \tag{9}$$

Table 1 compares drift velocities for various cases from the code with $I = 0$ to those from Eq. (9). In general, the agreement is very good.

Clearly, the gradient drift is more severe when B_w is a considerable fraction of B_o. However, if B_w is held constant at 1 kG and B_o increased to 4 kG, $v_d(\gamma = 2.2)$ is not significantly reduced. This relative insensitivity to B_o is due to the increased $v_\perp$, and hence a larger v_d, as B_o approaches the cyclotron resonance field B_r. One consequence of this large drift near resonance is that it limits the degree of gain enhancement achievable through the magneto-resonance effect.[4,16]

If B_0 sufficiently exceeds B_r, the drift can be quite small. For $\gamma = 2.2$ and $\lambda_w = 3$ cm, the resonant magnetic field $B_r \approx 7$ kG. With $B_o = 8$ kG and $B_w = 1$ kG, the particle still drifts to the wall as shown in Fig. 4a. The drift is now in the opposite direction to that below resonance since $v_\perp$ changes sign when $B_z > B_r$. This has the effect of changing the phase of v_y oscillations with respect to those of B_z; i.e., v_y is positive (for $x > 0$) when B_z is a minimum, so that the particle drifts in the +y direction. When B_o is increased to 10 kG for this case, the drift is small enough that the electron remains confined for > 30 periods as shown in Fig. (4b). The confinement remains very good when the self fields of a 500 A beam are added.

In principle, operation of FEL experiments with $B_o > B_r$ is possible and has been demonstrated with a high current, $\gamma \approx 3.5$ beam in a helical wiggler.[4] However, competing processes such as the cyclotron maser[15] interaction can produce large radiated powers at frequencies close to those of the FEL interaction in this beam energy and magnetic field regime.

Table 1.

B_o(kG)	B_w(kG)	γ	$\frac{\Omega_o}{kv_z}$	X(cm)	$\frac{v_d}{c}$ (code)	$\frac{v_d}{c}$ (Eq. 9)
2	1	2.2	.29	.4	.047	.050
4	1	2.2	.57	.4	.042	.046
2	.5	2.2	.29	.4	.011	.013
2	1	2.2	.29	.2	.019	.022
2	1	3.0	.21	.4	.029	.034
4	1	3.0	.42	.4	.019	.022
4	.5	3.0	.42	.4	.0051	.0056
4	1	10.0	.11	.4	.0043	.0049
10	5	10.0	.28	.4	.057	.056

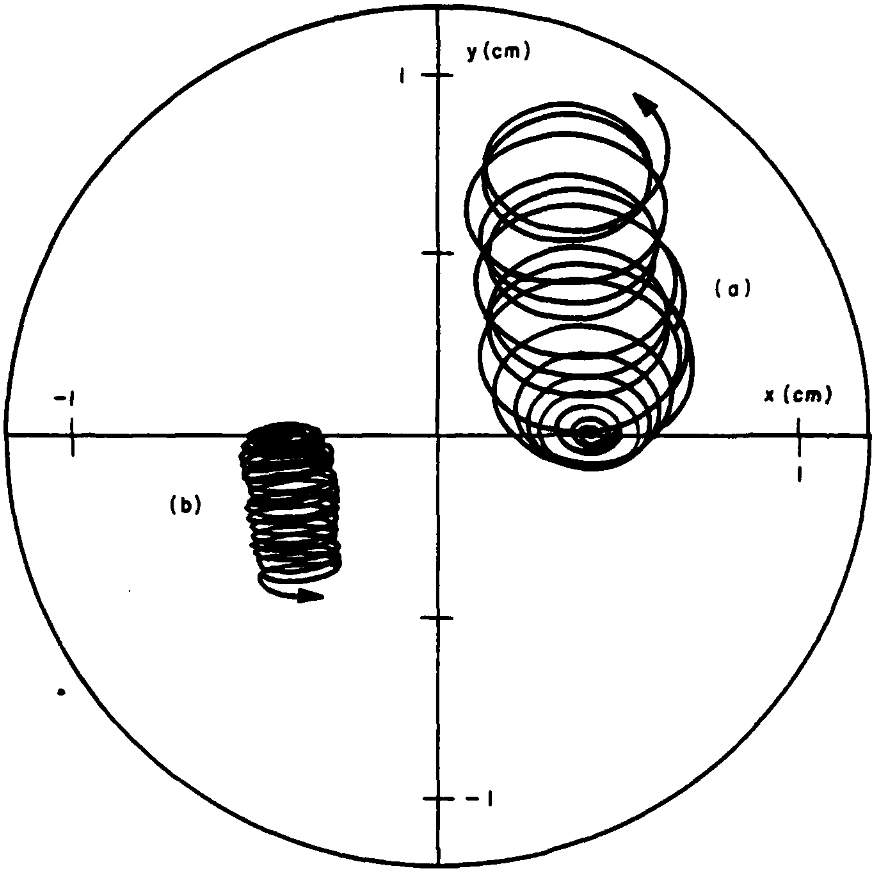

Fig. 4 - Electron trajectories in linear wiggler with I = 0, γ = 2.2, B_w = 1 kG, and r_o = .4 cm. a) B_o = 8 kG, b) B_o = 10 kG.

Consequently, analysis of experimental results is more difficult. Also, arbitrarily large guide fields are not possible with a magnetically shielded diode simply because the electrons will be mirrored by the field. The equilibrium beam radius[17] in such a case depends only on γ, I, B_o, and the beam emittance ε, so for particular beam parameters suitable values of B_o are limited. The required field[18] in kG for a matched beam radius R in cm can be written

$$B^2 = \frac{1.36\ I}{R^2 \beta\gamma} + \frac{11.56\ \varepsilon_n^{\ 2}}{R^4}\ , \tag{10}$$

where I is in kA and ε_n is the normalized emittance in rad cm. For example, if $\varepsilon_n = .14\ \pi$ rad-cm (about the lowest value expected for a thermionic cathode beam with I ≃ 750–1000 A)[18], a $\gamma = 2.2$ beam with R = .3 (.5) cm requires $B_o = 5.5$ (2) kG for $I \lesssim 800$ A. In this regime, the beam is emittance dominated so that the required field is relatively insensitive to I. In principle, smaller radius beams could be used with larger B_o, thereby doubly reducing v_d. However, experimentally this is very difficult at high current levels.[17]

To analyze the effect of a shielded diode on propagation through the wiggler, we repeat the above calculations with an initial v_θ corresponding to $P_{\theta o} = 0$. Note that I = 1.75 kA in this case, which is the current required for constant radius propagation in only the solenoidal field with these initial conditions. As shown in Fig. 5, the electron propagates at nearly constant radius until the wiggler amplitude becomes large enough that the

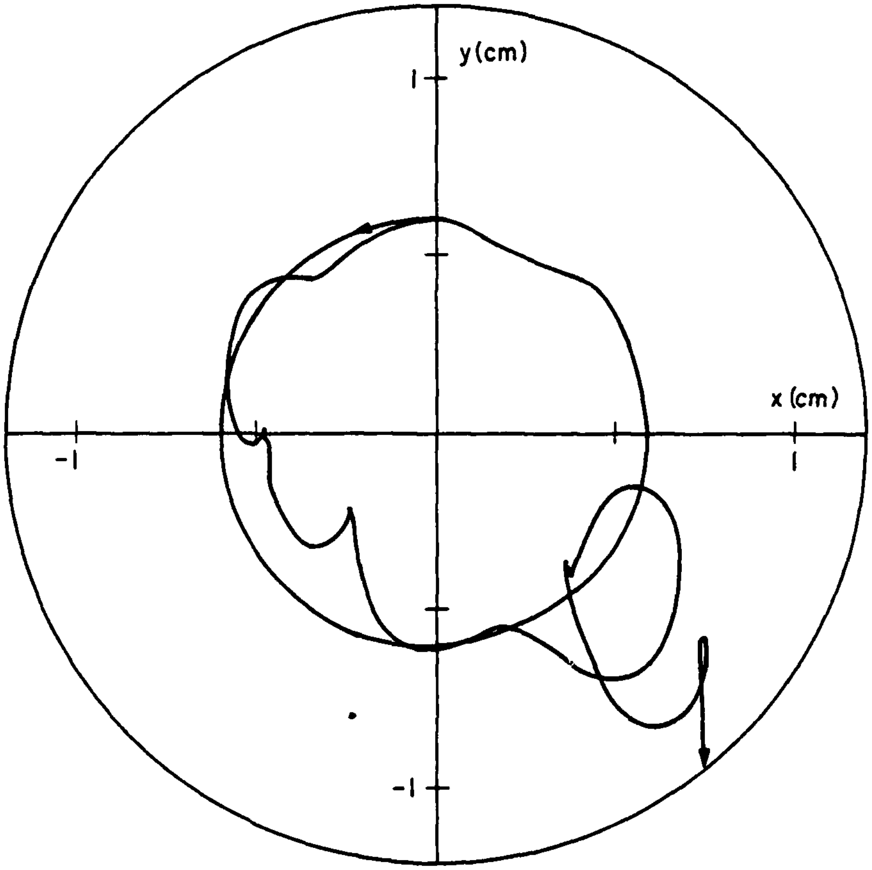

Fig. 5 - Electron trajectories in linear wiggler with $P_{\theta o} = 0$, $I = 1.75$ kA, $\gamma = 2.2$, $B_o = 2$ kG, $B_w = 1$ kG, $r_o = .6$ cm.

gradient drift begins to dominate. Then the particle is lost just as in the $v_{\theta o} = 0$ case.

The behavior of an off-axis electron in a linear wiggler and axial guide field should be compared to that when a helical wiggler is used.[13] We approximate the helical wiggler field by[6]

$$B_x^h = b(z)B_w\{-\sin kz\ [1 + \frac{1}{8}k^2(3x^2 + y^2)] + \frac{1}{4}k^2\ x\ y\cos kz\}$$

$$B_y^h = b(z)B_w\{\cos kz\ [1 + \frac{1}{8}k^2(x^2 + 3y^2)] - \frac{1}{4}k^2\ x\ y\sin kz\} \qquad (11)$$

$$B_z^h = -kb(z)B_w[1 + \frac{1}{8}k^2(x^2 + y^2)](x\cos kz + y\sin kz).$$

This expansion of the true Bessel function expression for B^h is valid for $kr < 1$. Friedland's treatment[13] of electron propagation in this case assumes a radially uniform wiggler, and his stability condition (Eq. 2a) is not stringent enough when the radial variation is included. For example, Friedland finds stable orbits when $\gamma = 1.587$, $\lambda_w = 4$ cm, $v_{\perp o} = 0$, $r_o = 0$, $B_o = 1.26$ kG, and $B_w = 1.04$ kG, so that Eq. 2a is barely satisfied, and we can duplicate his results if we remove the radial variation from B^h. However, with the B^h given in Eq. 11, we find that the wiggler field must be reduced to ~625 G to obtain stable orbits.

Although the radial dependence in B^h does narrow the allowable range of operating parameters, stable orbits with $\langle r\rangle \approx$ cons't are achievable with a helical wiggler in a guide field. Electron trajectories in a helical wiggler for the same conditions as used previously with a linear wiggler are shown in

Figs. 6 and 7. Figure 6 is to be compared to Fig. 2 and Fig. 7 to Fig. 5. In both cases, the electron is well-confined. It is interesting to note that in Fig. 6, the electron born at $(x,y) = (.4,0)$ initially $\vec{E} \times \vec{B}$ drifts in the + θ direction, but then reverses direction as the wiggler amplitude increases. Since grad B^h is radial, the grad B^h drift is in the $-\theta$ direction and does not lead to the beam expansion observed with the linear wiggler.

The linear wiggler drift described here imposes additional constraints on the parameters of an FEL experiment. Obviously, the beam radius must be kept as small as possible to minimize particle loss from the edge of the beam. Preliminary experimental results by our group with a field immersed, apertured source indicate that particle losses can be kept acceptably small in this way. Also, if γ is large enough that $B_r >> B_o >> B_w$ can be satisfied for relatively large B_w, then the drift can be kept small while achieving acceptably large v_w.

Finally, it should be noted that the drift arises from the asymmetry of the linear wiggler and the corresponding absence of focusing forces in the direction perpendicular to the wiggler field. Therefore, it should be possible to stabilize the drift by imposing an additional focusing force in that direction. For example, preliminary results indicate that electron propagation through a "square" or symmetrized linear wiggler is very stable. Such a wiggler has an additional component $B_y^\ell = \cosh ky \cos kz$ and a corresponding addition to B_z^ℓ of $-B_w \sinh ky \sin kz$. An electron trajectory through such a wiggler is shown in Fig. 8.

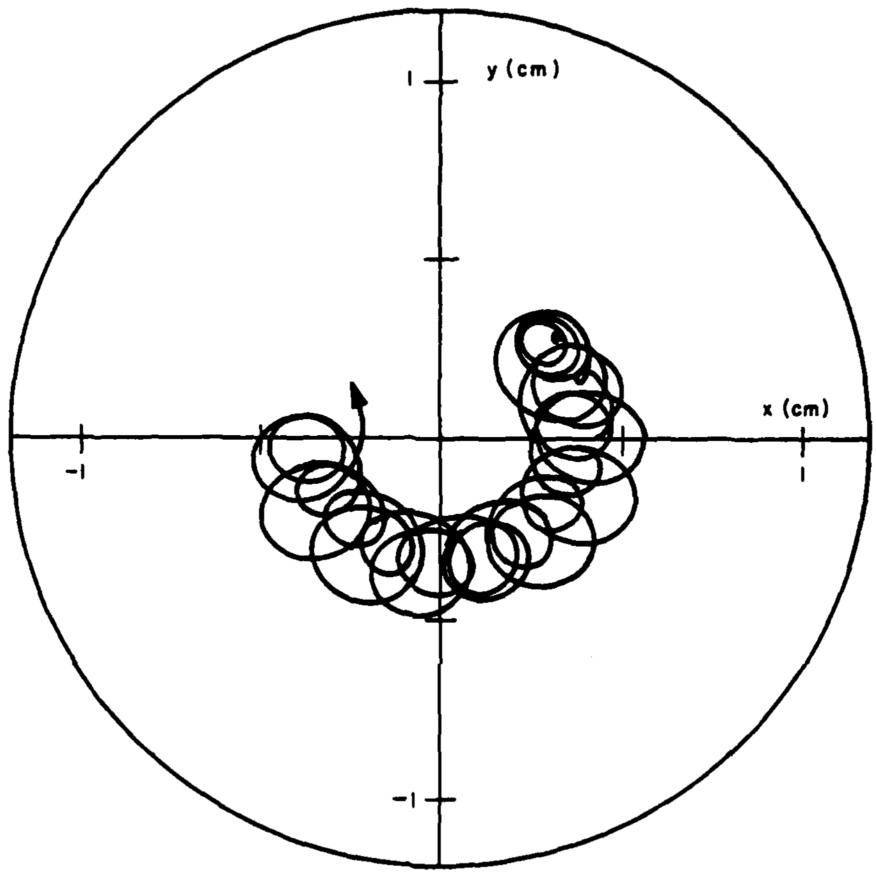

Fig. 6 - Electron trajectories with helical wiggler and $\gamma = 2.2$, $B_o = 2$ kG, $B_w = 1$ kG, $\lambda_w = 3$ cm, $I = 250$ A, $r_o = .4$ cm, and $v_{\perp o} = 0$.

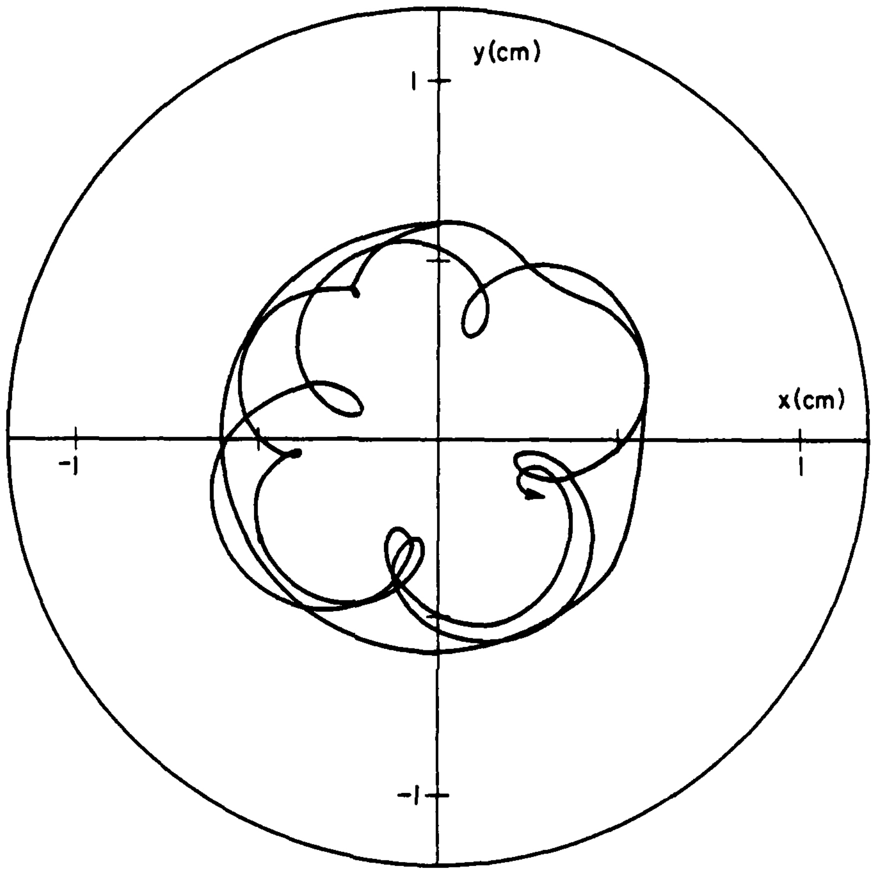

Fig. 7 - Electron trajectories in helical wiggler with $P_{\theta o} = 0$, $I = 1.75$ kA, $\gamma = 2.2$, $B_o = 2$ kG, $B_w = 1$ kG, and $r_o = .6$ cm.

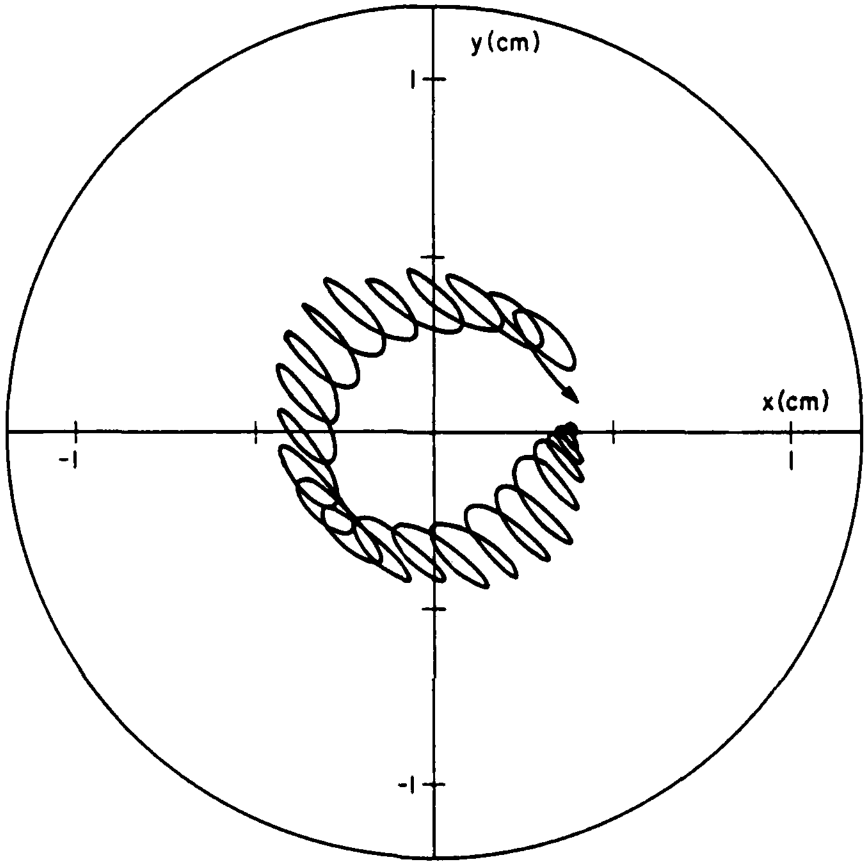

Fig. 8 - Electron trajectories in symmetrized linear wiggler with $I = 0$, $\gamma = 2.2$, $B_o = 2$ kG, $B_w = 1$ kG, $r_o = .4$ cm, and $v_{\perp o} = 0$.

In conclusion, a gradient drift has been shown to exist for a linear wiggler in an axial guide field. The drift can be substantial with small or large beam current in some parameter ranges, for a wide range of initial conditions. However, the advantages of a linear wiggler are sufficient in many cases to either limit operation to a "stable" parameter regime or to impose additional focusing forces to stabilize the drift.

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